Decelerator Fabric Constants Required by the Generalized Form of Hooke's Law

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The availability of the required elastic constants for use in structural analyses of fabric structures is exceedingly poor in comparison with the current situation for the common structural materials. Experiments with nylon-polyurethane-coated fabric have indicated highly nonlinear characteristics of the elastic constants of Hooke's law, a strong dependency of these coefficients on all three membrane loads, and a lack of symmetry in the Hookian relationships. A complete set of the nine coefficients of the generalized form of Hooke's law are furnished and analyzed for the test fabric.

Introduction

THE subject paper is on the second of two papers that are an outgrowth of a NASA contractual responsibility to furnish the material elastic constants for use in a structural analysis of a parawing. The first of these papers (Ref. 1) deals with the test apparatus and procedures for testing cylindrical fabric specimens for determining the stress-strain relationships from which the results of this paper have been derived.

Fabrics are being used widely as structural materials in the aerospace field. Hence, the engineer is faced with the problem of structural design, deformation, and stress analyses that intimately involve the fabric's macroscopic mechanical coefficients which are usually nonexistent. Also, there are four mechanical constants (coefficients of interaction of the first and second kinds[2]) that are significant to fabric deformation but can usually be neglected for the conventional structural materials. Furthermore, the material characteristics of woven fabrics are not constant and are generally nonlinear functions of the biaxial normal stresses and the accompanying shear stresses.

It is the purpose of this paper to provide the full set of Hookian coefficients associated with the stress-strain data of Ref. 1. These coefficients represent the macroscopic mechanical characteristics needed for a continuum approach to the structural analysis of a membrane fabric structure. The generalized Hooke's law for the two-dimensional anisotropic planar stress problem involves nine coefficients. For conconservative materials, six of these coefficients are independent and the remaining three are dependent. For fabric materials, it is clearly observed from tests and from reasoning that the straining is not conservative and nine elastic coefficients are found independent and necessary. The two-dimensional form of the generalized Hooke's law is indicated by the following matrix equation.

$$\begin{Bmatrix} \varepsilon_w \\ \varepsilon_f \\ \gamma \end{Bmatrix} = \begin{bmatrix} 1/E_w & -\mu_{w,f}/E_f & \eta_{w,wf}/G \\ -\mu_{f,w}/E_w & 1/E_f & \eta_{f,wf}/G \\ \eta_{wf,w}/E_w & \eta_{wf,f}/E_f & 1/G \end{bmatrix} \begin{Bmatrix} \sigma_w \\ \sigma_f \\ \tau \end{Bmatrix} \qquad (1)$$

The ε_w, ε_f, and γ are strains in the warp and fill directions and the shear deformation, respectively. Also, σ_w, σ_f, and τ are membrane stresses in the warp and fill directions and shear stress, respectively. The diagonal terms are reciprocals of Young's moduli and the shear modulus. The $\mu_{w,f}$ and $\mu_{f,w}$

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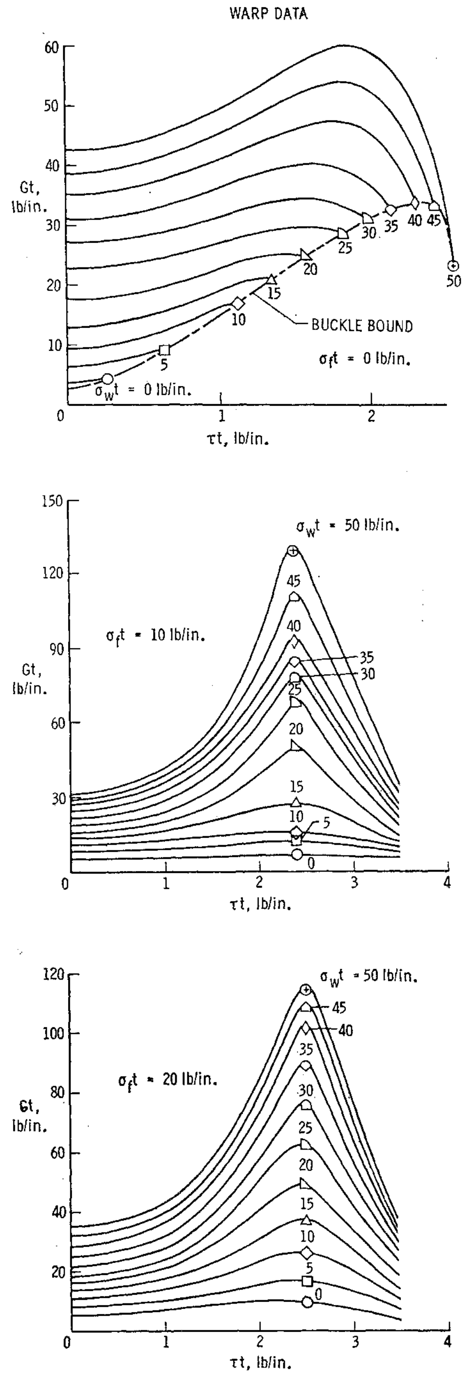

Fig. 1 Shear moduli.

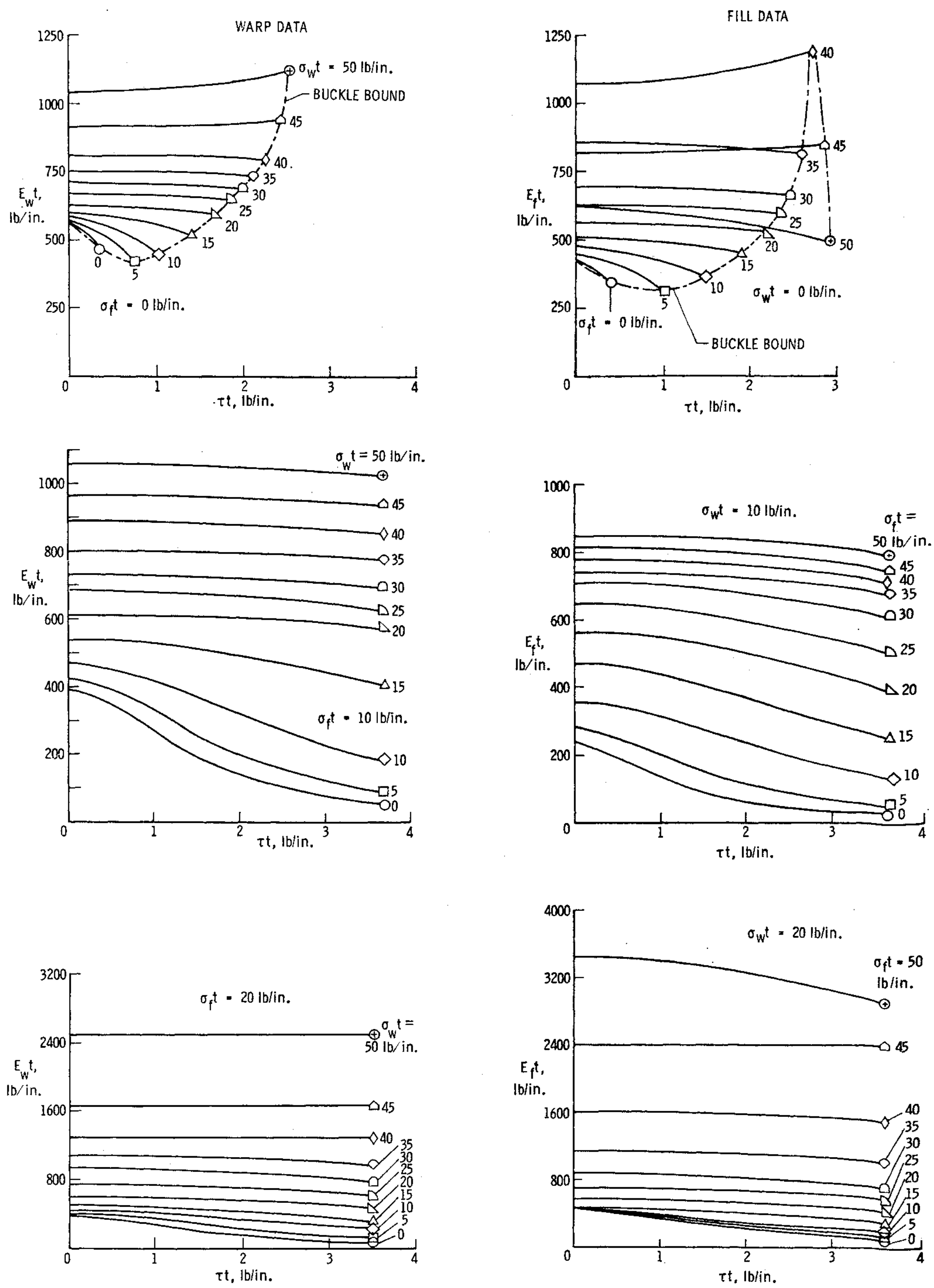

Fig. 2 Young's moduli.

are Poisson's ratios (contraction in the warp direction due to strain in the fill direction and contraction in the fill direction due to strain in the warp direction, respectively). The $\eta_{wf,w}$ and $\eta_{wf,f}$ coefficients of the last row and $\eta_{w,wf}$ and $\eta_{f,wf}$ coefficients of the last column are not too conventional and are defined as coefficients of interaction of the first and second kind, respectively, and relate shear strain to extensional strain and vice-versa, respectively. The coefficients of the "Generalized Hooke's Law," Eq. (1), can be related by the following.

$$\begin{aligned}
&\frac{1}{E_w}=\frac{\partial\varepsilon_w}{\partial\sigma_w}, \quad \frac{\mu_{w,f}}{E_f}=-\frac{\partial\varepsilon_{w,f}}{\partial\sigma_f}, \quad \frac{\eta_{w,wf}}{G}=\frac{\partial\varepsilon_w}{\partial\tau}\\
&\frac{\mu_{f,w}}{E_w}=-\frac{\partial\varepsilon_{f,w}}{\partial\sigma_w}, \quad \frac{1}{E_f}=\frac{\partial\varepsilon_f}{\partial\sigma_f}, \quad \frac{\eta_{f,wf}}{G}=\frac{\partial\varepsilon_f}{\partial\tau}\\
&\frac{\eta_{wf,w}}{E}=\frac{\partial\gamma}{\partial\sigma_w}, \quad \frac{\eta_{wf,f}}{E}=\frac{\partial\gamma}{\partial\sigma_f}, \quad \frac{1}{G}=\frac{\partial\gamma}{\partial\tau}
\end{aligned} \tag{2}$$

The preceding forms of the coefficients are applicable to the piecewise linear concept of Eq. (1). The symbols $\varepsilon_{f,w}$ and $\varepsilon_{w,f}$ are the strain in the fill and warp directions, respectively, due to strains in the warp and fill direction, respectively.

Material Coefficients

For brevity, the general comments that follow are restricted only to the results of test where the warp material is parallel with the longitudinal axis of the cylindrical test specimen. Nevertheless, the comments are equally applicable to the fill data by simply switching the subscripts. Test data were reduced by "least square" fits to the experimental data such that a generalized function $X=f(\sigma_w t, \tau t)$ is defined analytically for fixed values of the third independent variable $\sigma_f t$. The generalized variable X represents the strains ε_w, ε_f, $\varepsilon_{w,f}$, $\varepsilon_{f,w}$, and γ from which the desired material coefficients are related by the partial derivatives shown by Eq. (2).

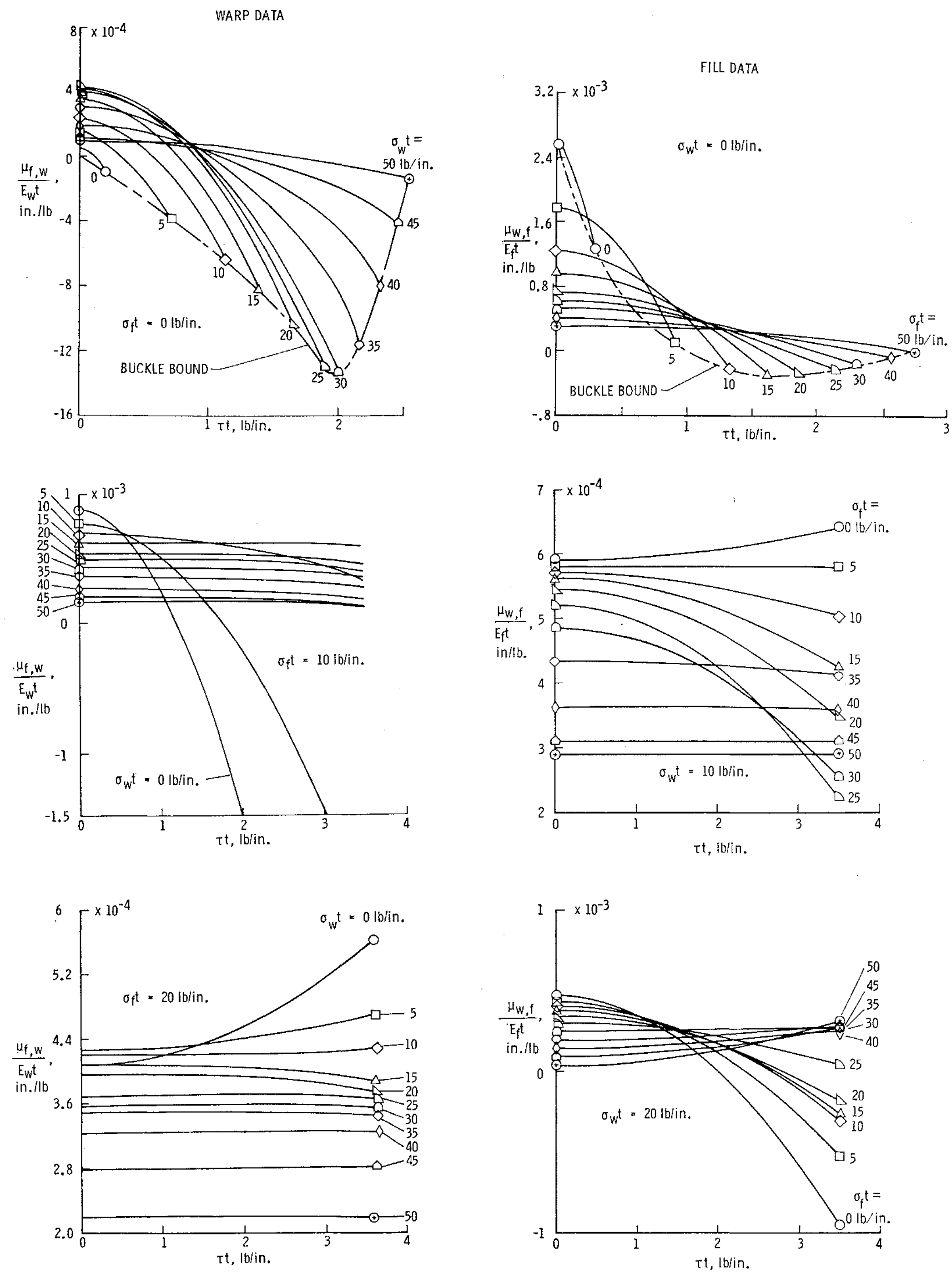

Fig. 3 Functions of Poisson's ratio.

Experimental data on a 2.25 oz nylon polyurethane-coated fabric have been processed in accordance with the foregoing procedures. Data for the off-diagonal coefficients and reciprocals of the diagonal coefficients of Eq. (1) are submitted on Figs. 1–5. It is expedient to maintain the thickness t with the coefficients since they are obtainable directly as such. The accuracy and validity of these types of data are influenced by the rate of loading, incipient buckling, level of axial load, uploading or downloading, loading sequence, history, and numerous other factors. The E_wt, Gt, and $\mu_{f,w}/E_wt$ data from the warp tests are symmetrical with respect to their ordinate axes. Hence, they are the same for both positive and negative values of τt. The $\eta_{wf,w}/E_wt$ and $\eta_{w,wf}/Gt$ data are antisymmetric about their ordinate axes, and, thus, are of opposite signs for negative values of τt. For the surfaces E_wt and Gt having negative values of σ_wt, these coefficients are set to zero. For the surfaces $\mu_{f,w}/E_wt$, $\eta_{wf,w}/E_wt$, and $\eta_{w,wf}/Gt$ having negative values of extensional stresses, the coefficients are set to minus infinity. Buckling for the warp tests was not observed within the range of test values for the circumferential loadings (σ_ft) of 10 and 20 lb/in., although for the zero axial load curves ($\sigma_wt = 0$) excessive but stable shear deformation was evident.

On inspection of the Hookian relationship [Eq. (1)] it is noted that only one Gt coefficient is required among the nine material characteristics. The Gt coefficient can be found from both fill and warp test data and for ideal conditions should be identical. The comparative Gt values from the two processes are characteristically similar and quantitatively in reasonable agreement; hence only the warp data are shown.

By use of the E_wt data, it is easy to isolate $\mu_{f,w}$ from the ratio $\mu_{f,w}/E_wt$ for specific states of stress. Typical values of $\mu_{f,w}$ and $\mu_{w,f}$ from the Poisson's ratio functions for various combinations of the independent variables are shown in Table I. The value of $\mu_{f,w}$ approaches zero for incipient

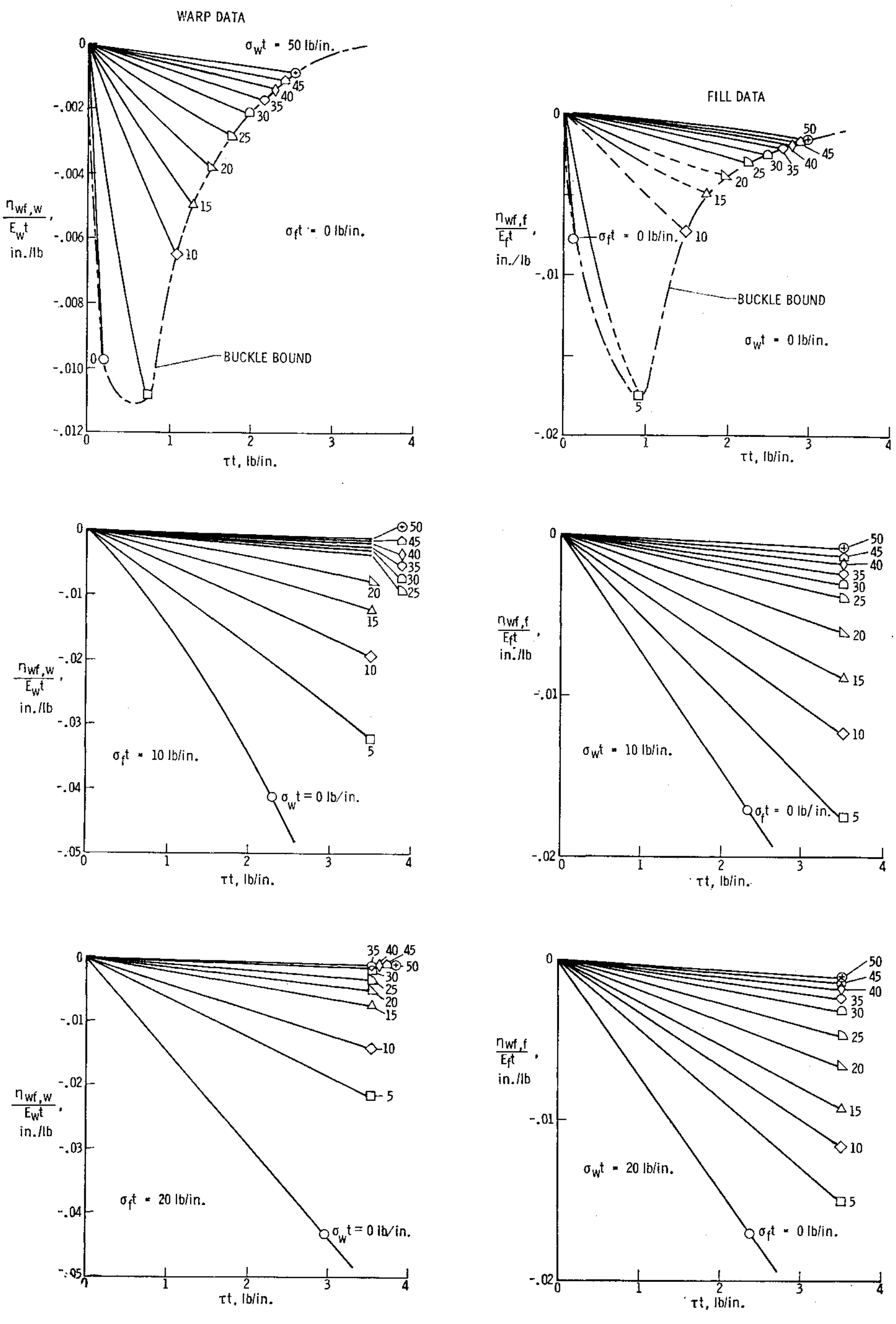

Fig. 4 Elastic constants involving the coefficients of interaction of the first kind.

Table 1 Typical values of Poisson's ratio

	$\sigma_f t = 10$ lb/in.					
	μ_{fw}			μ_{wf}		
$\tau t =$	0	1.0	2.0	0	1.0	2.0
$\sigma_w t = 10$	0.34	0.28	0.19	0.20	0.18	0.13
$\sigma_w t = 20$	0.33	0.33	0.32	0.18	0.14	0.06
	$\sigma_f t = 20$ lb/in.					
	μ_{fw}			μ_{wf}		
$\tau t =$	0	1.0	2.0	0	1.0	2.0
$\sigma_w t = 10$	0.18	0.18	0.14	0.31	0.29	0.24
$\sigma_w t = 20$	0.24	0.24	0.24	0.20	0.07	0.10

buckling and approaches minus infinity when fully buckled. The significance of the negative Poisson's ratio is explainable. In torsional buckling, the cylindrical specimen twists and warps extensively and reduces appreciably in circumference. As the axial load $\sigma_w t$ is increased, the twist is removed and the circumference begins to increase producing an increase in the normal strain (ε_f) which is opposite from the usual Poisson's ratio effect. It is suspected that the aforementioned phenomenon leading to negative Poisson's ratios is peculiar to the cylindrical test specimen under large twisting and would not be as evident from a planar test fixture.

The coefficients of interaction of the first and second kinds are symmetrically positioned in the Hookian relationship, and for fabrics, they do show similar variations as well as similar magnitudes. It is speculated that the ideal behavior of the coefficients of interaction of the first and seconds is to

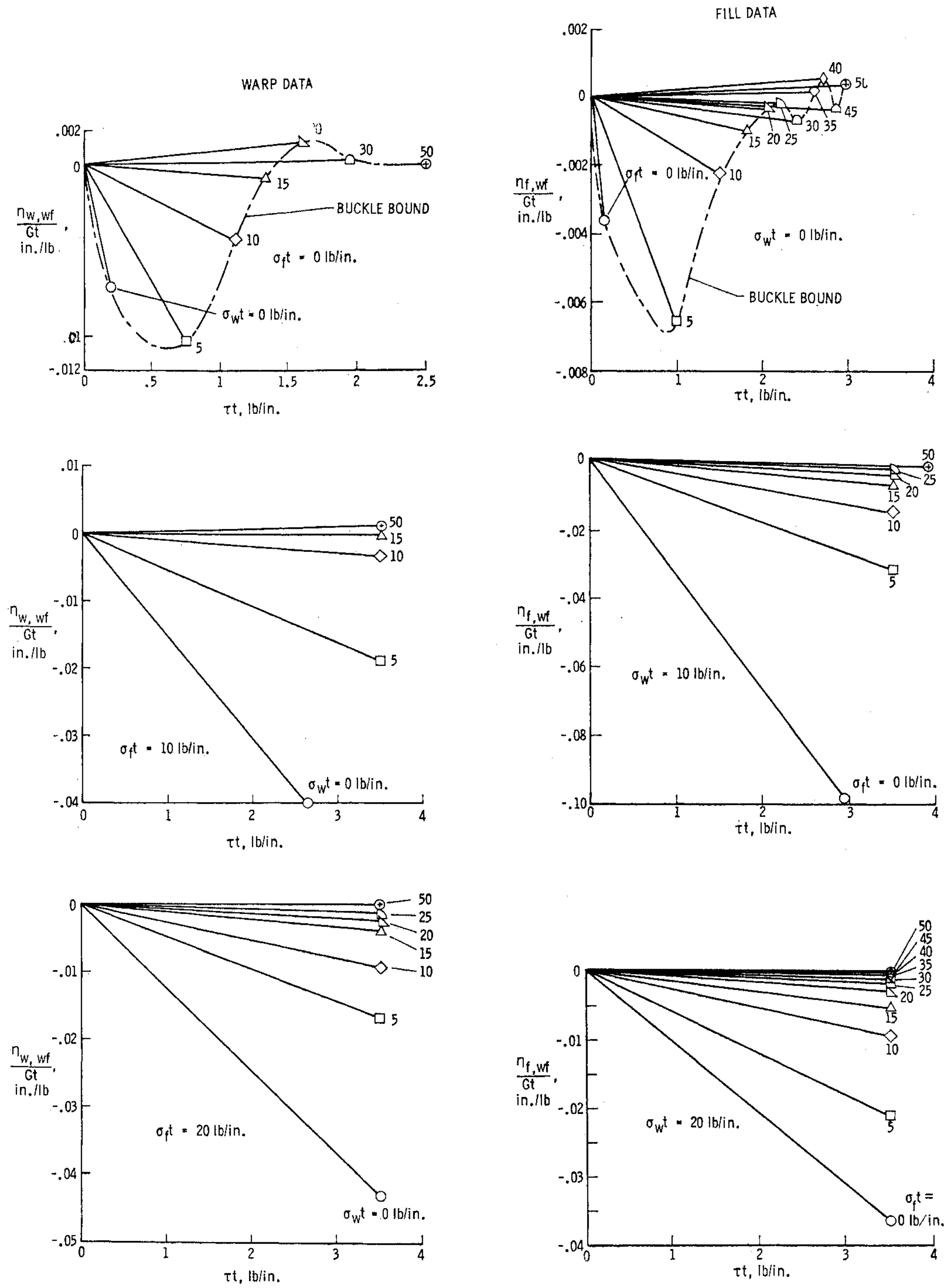

Fig. 5 Elastic constants involving the coefficients of interaction of the second kind.

exhibit a proportional relationship to τt, to be everywhere negative, and approach zero asymptotically for large values of the extensional stresses. It is seen on Fig. 5 that small positive excursions of the data occured. This is attributed to erratic data and undesirable features in the data fitting. Their relative value to other coefficients is indicated by the following numerical evaluation of the Hookian relationship for a typical membrane force vector.

$$\begin{Bmatrix} \varepsilon_w \\ \varepsilon_f \\ \gamma \end{Bmatrix} = \begin{bmatrix} 0.0043 & -0.0052 & -0.0028 \\ -0.00045 & 0.0059 & -0.012 \\ -0.016 & -0.01 & 0.059 \end{bmatrix} \begin{Bmatrix} 10 \\ 10 \\ 3 \end{Bmatrix} =$$

$$\begin{bmatrix} (\ 0.043 & -0.052 & -0.0084) \\ (-0.0045 & 0.059 & -0.036) \\ (-0.16 & -0.15 & 0.18) \end{bmatrix}$$

It is apparent from examination of the relative numbers that the functions involving the coefficients of interaction of both the first and second kinds are appreciable contributors to the strains.

Further Research

The materials study has revealed needed research as to methodology for structural analyses to accommodate asymmetric Hookian relationships. New test procedures and innovations in data processing are seen necessary to improve accuracy of both the acquired and processed data. Also, comparison of coefficients acquired from both planar and cylindrical test specimens would be a valuable investigation. The study of the two-dimensional material has indicated that similar effort should be considered for three-dimensional anisotropic filamental materials. In particular, examination of the importance of the off-diagonal coefficients for exotic new boron filament composites is needed.

Conclusions

An evaluation of the material coefficients for a 2.25-oz nylon fabric with polyurethane coating has resulted in the quantization of nine independent nonlinear coefficients that have not otherwise been investigated and assessed. The measured data reveal the coefficient matrix of the stress-strain relationship to be unsymmetric and the material coefficients to be nonlinear functions of the biaxial membrane loads and accompanying shear load.

References

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